UNDERGRADUATE FOURTH SEMESTER (HONOURS) EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 42115**

Course Code: SH/MTH/405/SEC-2 **Course Title: Graph Theory**

Time: 2 hours **Full Marks: 40**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning

1. Answer *any five* of the following questions: **(2X5=10)**

a) Is the degree sequence (5,3,3,3,2,2,1,1) graphical?

b) Show that on a digraph the total sum of in-degrees is equal to total sum of out-degrees.

c) Find the number of vertices in a graph with 15 edges, if each vertex has degree 2.

d) Show that a k-regular graph of order $2k-1$ is Hamiltonian.

e) Prove that a complete graph with n vertices contains $n(n-1)/2$ edges.

f) Check whether the complete bipartite graph$K_{2,4}$ isEulerian or Hamiltonian?

g) Show that in a connected graph of order n and size m ($m < n$), there exists atleast one pendent vertex.

h) Draw a graph whose adjacency matrix is given by

$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. Is this graph bipartite ?

2. Answer *any four of* the following questions: **(5X4=20)**

a) Show that a tree with n vertices has exactly $n-1$ edges.

b) (i) If a graph contains exactly two vertices of odd degree then show that there exists a path between these two vertices.

(ii) If G is simple with minimum vertex degree $\geq \frac{n-1}{2}$, then prove that$G$ is connected.

2+3 = 5

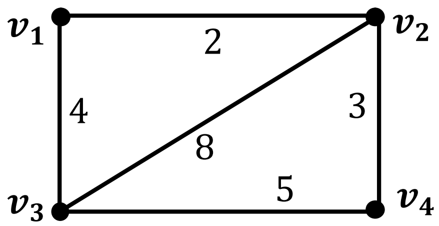
c) (i) If a simple graph G has at most $2n$ vertices and the degree of each vertex is at least n, then show that the graph is connected.

(ii) Let G be a graph and u, v be two vertices in G such that $u \neq v$. If there is a trail from $u$to v, then show that there is a path from u to v. **3+2=5**

d) (i) Define a Hamiltonian cycle.

(ii) Let u and v be two vertices of a connected simple graph G such that $d(u) + d(v) \geq n$. Then G is Hamiltonian if and only if $G + \{u, v\}$ is Hamiltonian. **1+4**

e) Define a weighted graph. Using Warshall's algorithm, find the distance between each pair of vertices of the following weighted graph



f) (i) Define the eccentricity of a vertex in a graph.

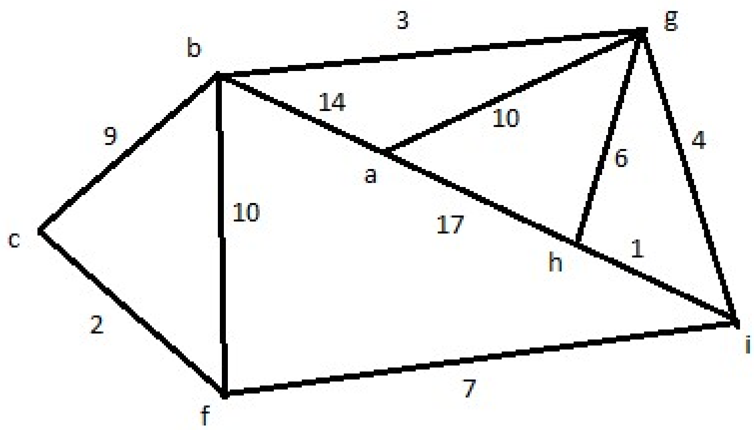
(ii) A person has to visit four cities $\{A, B, C, D\}$ starting from A and return to A after visiting all the cities exactly once. Find the cost saving optimal route where the travelling cost matrix among the cities is given below: **1+4**

	A	***B***	***C***	***D***
A	-	5	2	3
B	2	-	4	3
C	2	4	-	7
D	3	3	7	-

3. Answer *any one* of the following questions: (10X1=10)

a) (i) Show that on a bipartite graph every circuit is of even length.

(ii) Using Dijkstra's Algorithm find the length of the shortest path of the following graph from the vertex a to each of the vertices c, f and i.



(iii) Show that every connected graph has a spanning tree. **3+5+2**

b) (i)Show that the degree of a vertex is invariant under graph isomorphisms.

(ii) Define a semi-Eulerian graph and draw a semi-Eulerian graph which is not Eulerian.

(iii)Show that a simple graph (order ≥2) has atleast two vertices of the same degree.

(iv) Let $I(G) = (a_{ij})_{n\times m}$ be the incidence matrix of a graph G with ordered vertex set $\{v_1, v_2, \ldots, v_n\}$. Show that

$$\sum_{j=1}^{m} a_{ij} = d(v_i).$$

3+2+3+2

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